TonyCrane 普通物理学 I (H)

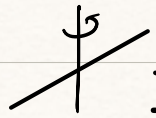
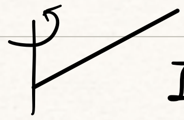
不确定度 $\Delta f^2 = \left(\frac{\partial f}{\partial x}\Big|_{\langle x\rangle\langle y\rangle}\right)^2 \Delta x^2 + \left(\frac{\partial f}{\partial y}\Big|_{\langle x\rangle\langle y\rangle}\right)^2 \Delta y^2$

$\langle x\rangle$ 平均 $\int p(x)\,x\,dx$ $\Delta x^2 = \langle (x-\langle x\rangle)^2\rangle$

Torque 力矩. $\tau = rF\sin\varphi$

Moment of inertia 转动惯量 $I = mr^2$ $\Sigma\tau = I\alpha$

Parallel-axis theorem. $I' = I + md^2$ I'为绕实际轴, d为两轴间距

 $I = \frac{1}{12}ML^2$  $I = \frac{1}{3}ML^2$

实心球 $I = \frac{2}{5}MR^2$

薄空心球 $I = \frac{2}{3}MR^2$

圆环柱 $I = \frac{1}{2}M(R_1^2 + R_2^2)$ 长宽ab绕中心. $I = \frac{1}{12}M(a^2+b^2)$

简谐运动 $d^2x/dt^2 = -\omega^2 x$ $x = A\cos(\omega t + \varphi)$

振幅 amplitude A 角频率 angular freq. ω

频率 frequence $f = \frac{1}{T} = \frac{\omega}{2\pi}$ 周期 period T

$K = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{x}^2$ $U = \frac{1}{2}kx^2$ $E = K + U = \frac{1}{2}kA^2$ $\omega = \sqrt{\frac{k}{m}}$

阻尼振动 damped oscillation $m\,d^2x/dt^2 = -kx - b\,dx/dt$

$x = Ae^{-\frac{b}{2m}t}\cos(\omega t + \varphi)$ $\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

受迫振动 forced oscillation $F_{ext}\cos\omega t - kx - b\,dx/dt = m\,d^2x/dt^2$

$x = Ae^{-\frac{b}{2m}t}\cos(\omega' t + \varphi') + A\cos(\omega t + \varphi)$ $\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$

$A = \frac{F_{ext}/m}{\sqrt{(\omega^2 - \omega_0^2)^2 - \gamma^2\omega^2}}$ $\omega_0 = \sqrt{k/m}$ $\gamma = b/m$

杨氏模量 $Y = \frac{F/A}{\Delta L/L_i}$ Shear modulus: $S = \frac{F/A}{\Delta x/h}$

Bulk modulus $B = -\frac{\Delta F/A}{\Delta V/V_i}$

线性波动方程 $\partial^2 u/\partial t^2 = v^2\,\partial^2 u/\partial x^2$ $v = a\sqrt{k/m}$

距平衡位置位移 平衡位置间距

正弦波 $y = A\sin(kx - \omega t + \varphi)$ $\omega = vk$ $k = \frac{2\pi}{\lambda}$

$dE = dK + dU = \frac{1}{2}\mu\omega^2A^2dx$ $P = \frac{dE}{dt} = \frac{1}{2}\mu\omega^2A^2v$

(全偶数倍)

$\Delta r = \frac{\varphi}{2\pi}\lambda \quad \varphi = 2n\pi$ 增强 $\quad \varphi = 2(n+1)\pi$ 减弱

Beating $\quad y_1 = A\cos\omega_1 t = A\cos 2\pi f_1 t \quad y_2 = A\cos\omega_2 t = A\cos 2\pi f_2 t$

$$y = y_1 + y_2 = \left(2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t\right)\cos 2\pi\left(\frac{f_1+f_2}{2}\right)t$$

$$A' = 2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)$$

Standing waves $\quad y_1 = A\sin(kx-\omega t) \quad y_2 = A\sin(kx+\omega t)$

$$y = y_1 + y_2 = (2A\sin kx)\cos\omega t$$

node: $\quad kx = n\pi \quad x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \cdots$

antinode: $\quad kx = (n+\frac{1}{2})\pi \quad x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \cdots$

$\beta = \frac{1}{10}\log\left(\frac{\Delta P}{\Delta P_{ref}}\right)^2 \quad \Delta P_{ref} = 2\times 10^{-5}\ N\cdot m^{-2}$

$I = \frac{1}{2}\rho v(\omega s_{max})^2 \quad \beta = \frac{1}{10}\log\left(\frac{I}{I_0}\right) \quad I_0 = 1\times 10^{-12}\ W/m^2$

Doppler Effect $\quad f' = \frac{c \pm v_0}{c \mp v_s} f$

$$w = \frac{u+v}{1+\frac{u}{c}\frac{v}{c}}$$

Lorentz transformation $\quad K \to K'$

$$\begin{cases} ct' = \gamma(ct - \beta x) \\ x' = \gamma(x - \beta ct) \\ y' = y \\ z' = z \end{cases} \qquad \beta = v/c \quad \gamma = 1/\sqrt{1-\beta^2}$$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$(\Delta s)^2 = -c^2(\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2} \quad u_y' = \frac{u_y}{\gamma(1 - u_x v/c^2)} \quad u_z' = \frac{u_z}{\gamma(1 - u_x v/c^2)}$$

$$f = \sqrt{\frac{1+v/c}{1-v/c}}\, f_0 \quad m' = \frac{m}{\sqrt{1-v^2/c^2}} \quad p = \gamma m u$$

$$K = \gamma mc^2 - mc^2 \quad E_0 = mc^2$$

$PV = nRT \quad n = 8.315\ J/mol\cdot K$

$PV = \frac{N}{N_A}RT = Nk_BT \quad k_B = 1.38\times10^{-23}\ J/K$

$\ln V = \ln T + \ln(nR/p)$

$\beta = \left(\frac{1}{V}\frac{dV}{dT}\right)_p = \left(\frac{d\ln V}{dT}\right)_p = \frac{d\ln T}{dT} = \frac{1}{T}$

$\left(p + \frac{aN^2}{V^2}\right)(V - Nb) = Nk_BT \quad T\to 0\ V = b \quad p \to p + \frac{aN^2}{V^2}$

$p = \frac{Nk_BT}{V - bN} - a\frac{N^2}{V^2}$ Critical case : $\frac{\partial p}{\partial V} = \frac{\partial^2 p}{\partial V^2} = 0$

$\Rightarrow N/V = 1/3b \quad k_BT_c = \frac{8}{27}\frac{a}{b}$

$T_C = T - 273.15 \quad T_F = \frac{9}{5}T_C + 32^\circ F$

Linear expansion $\Delta L = \alpha L_0 \Delta T$

Volume expansion $\Delta V = \beta V_0 \Delta T \quad \beta = 3\alpha$

Ideal gas : $n^{-\frac{1}{3}} >> a \quad k_BT >> \varepsilon_{int} \quad K/N \sim k_BT \quad E_{int}/N \sim \varepsilon_{int}$

$U = \frac{f}{2}Nk_BT \quad C_V = \frac{dU}{dT} = \frac{f}{2}Nk_B$ molar $C_V^{mol} = \frac{f}{2}R$

Boltzmann distr. $n = n_0 e^{-mgy/k_BT}$

Maxwell distr. $N(v)\,dv = N\left(\frac{m}{2\pi k_BT}\right)^{3/2} e^{-\frac{mv^2}{2k_BT}} 4\pi v^2 dv$

$\underbrace{\int_0^{+\infty} e^{-ax^2}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}}_{=I_0} \quad \int_0^{+\infty} xe^{-ax^2}dx = \frac{1}{2a} = I_1 \quad \int_0^{+\infty} x^2e^{-ax^2}dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}} = I_2$

$I_{2n} = (-1)^n\frac{d^n}{da^n}I_0 \quad I_{2n+1} = (-1)^n\frac{d^n}{da^n}I_1 \quad 3\to\frac{1}{2a^2} \quad 4\to\frac{3}{8}\sqrt{\frac{\pi}{a^5}}$

$v_{mp} = \sqrt{\frac{2kT}{m}}$ 最概然速率. $dN(v)/dv = 0$

$v_{rms} = \sqrt{\frac{3kT}{m}}$ 方均根速率 $v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{\int_0^{+\infty} v^2N(v)\,dv}{N}}$

$\bar{v} = \sqrt{\frac{8kT}{\pi m}}$ 平均速率 $\bar{v} = \frac{1}{N}\int_0^{+\infty} vN(v)\,dv$

$Q = mL$ L: Latent heat 潜热 J/kg $\quad Q = cm\Delta T$

$\Delta U = Q - W$ 1st law of thermodyn.

Fourier heat conduction law : $P = \frac{Q}{\Delta t} = -\kappa_T A\frac{dT}{dx}$

mean free path 平均自由程. 两次碰撞间平均距离

$l=\frac{\bar{v}t}{z}=\frac{\bar{v}t}{n_v\pi d^2\bar{v}t}=\frac{1}{n_v\pi d^2}=\frac{k_BT}{\pi d^2 p}$ $p=n_vk_BT$ $n_v=\frac{p}{k_BT}$

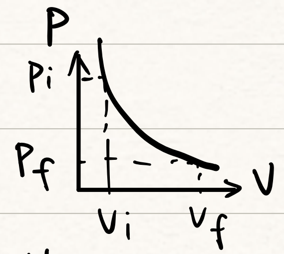
算相对运动 $v\to\sqrt{2}v$ $l=k_BT/\sqrt{2}\pi d^2 p$

碰撞频率 $f=\bar{v}/l$

Quasi-static 准静态、

Isothermal processes 等温过程

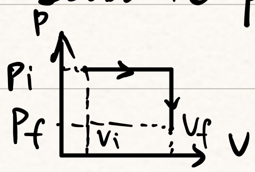
Isothermal expansion $PV=C$



$W=\int_{V_i}^{V_f}PdV=\int_{V_i}^{V_f}\frac{Nk_BT}{V}dV=Nk_BT\ln\frac{V_f}{V_i}$ $\Delta U=0$ $Q=W$

Adiabatic processes 绝热过程 Isobaric / Isovolumetric (Isochoric) 等压/容

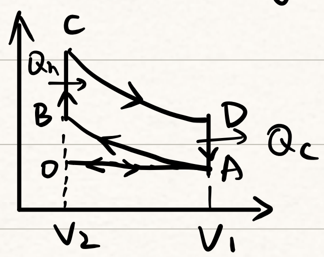
Isobaric processes



$W=P(V_f-V_i)$ $\Delta U=C_V\Delta T$ $Q=\Delta U+W=C_V\Delta T+P\Delta V$

$Q=C_V\Delta T+Nk_B\Delta T$ $C_P=\left(\frac{Q}{\Delta T}\right)_P=C_V+Nk_B=\frac{f+2}{2}Nk_B$

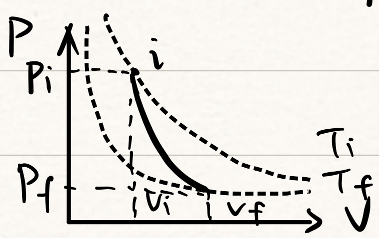
Gasoline Engine 汽油机 Otto cycle



$Q_h=nC_V(T_C-T_B)$ $Q_c=nC_V(T_D-T_A)$

$e=\frac{W}{Q_h}=\frac{Q_h-Q_c}{Q_h}=1-\frac{T_D-T_A}{T_C-T_B}$

Adiabatic expansion



$dQ=0$ $C_VdT=dU=-PdV$

$C_VdT/T=-Nk_BdV/V$ $TV^{\gamma-1}=C$ $PV^{\gamma}=C$

$\gamma=C_P/C_V=1+Nk_B/C_V$

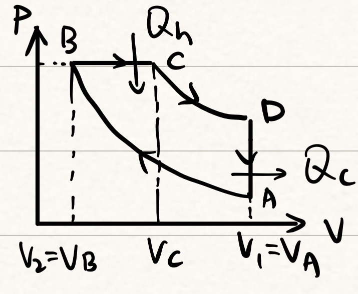
A→B C→D 为绝热过程. $T_AV_1^{\gamma-1}=T_BV_2^{\gamma-1}$ $T_DV_1^{\gamma-1}=T_CV_2^{\gamma-1}$

$(V_2/V_1)^{\gamma-1}=T_A/T_B=T_D/T_C$ $e=1-T_A/T_B=1-T_D/T_C=1-(V_2/V_1)^{\gamma-1}$

Diesel Engine 柴油机 $r=V_A/V_B$ $r_c=V_C/V_B$

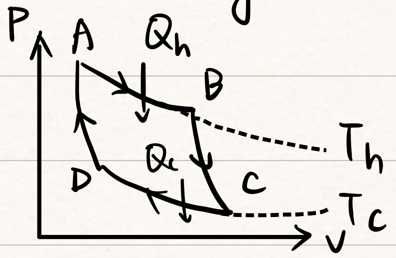
$T_A/T_B=(V_B/V_A)^{\gamma-1}=r^{-(\gamma-1)}$ $T_C/T_B=V_C/V_B=r_c$

$T_D/T_C=(V_C/V_D)^{\gamma-1}=(r_cV_B/V_A)^{\gamma-1}=(r_c/r)^{\gamma-1}$



$Q_h = nC_p(T_C - T_B)$ $Q_c = nC_v(T_D - T_A)$ $W = Q_h - Q_c$

Carnot Engine 卡诺热机 2等温+2绝热 且可逆



$Q_h = W_{AB} = nRT_h \ln\frac{V_B}{V_A}$

$Q_c = |W_{CD}| = nRT_c \ln\frac{V_C}{V_D}$

$T_hV_B^{\gamma-1} = T_cV_C^{\gamma-1}$ $T_hV_A^{\gamma-1} = T_cV_D^{\gamma-1}$

$\Rightarrow Q_c/Q_h = T_c/T_h$ $e = 1-\frac{Q_c}{Q_h} = 1-\frac{T_c}{T_h}$

违反第二定律

卡诺定律：在 T_c T_h 间没有比卡诺热机效率更高的热机

可逆热机 $Q_c/T_c = Q_h/T_h$ $\oint \frac{\text{đ}Q}{T} = 0$

$S(A) = \int_0^A \frac{\text{đ}Q}{T}$ $dS = \text{đ}Q/T$ $\text{đ}Q = TdS$ đQ沿可逆路径

$(\partial U/\partial V)_T = T(\partial P/\partial T)_V - P$

$p(T,V) = RT/V$ $U(T) = C_V^{mol}T = \frac{fRT}{2}$ $dS = \frac{1}{T}(dU + pdV) = \frac{1}{T}C_V^{mol}dT + \frac{R}{V}dV$

$S(T,V) = S_0 + C_V^{mol}\ln\frac{T}{T_0} + R\ln\frac{V}{V_0}$